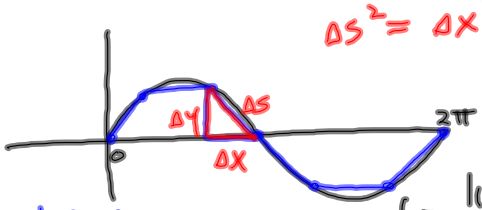
7.4 Length of a smooth curve

Approximate the length of the curve y = sin(x) from x = 0 to x = 2pi



$\Delta s^2 = \Delta x^2 + \Delta y^2$

$\Delta s = \sqrt{\Delta x^2 + \Delta y^2}$

$L = \lim_{\Delta x \to 0} \sum \sqrt{\Delta x^2 + \Delta y^2}$

definition:

$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum f(x)\Delta x$

$L = \lim_{\Delta x \to 0} \sum \sqrt{\left(1 + \frac{\Delta y^2}{\Delta x^2}\right)\Delta x^2}$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

$L = \lim_{\Delta x \to 0} \sum \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$

$\int_0^{2\pi} \sqrt{1 + (\cos x)^2}\,dx$

$L = \lim_{\Delta x \to 0} \sum \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,\Delta x$

7.6404

$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

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Definition of arclength

$\frac{dy}{dx}$ must be defined on $[a,b]$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

when $y = f(x)$

a, b: limits for x

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

when $x = g(y)$

c, d - limits for y

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Find the exact length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \le x \le 1$

$\frac{dy}{dx} = \frac{4\sqrt{2}}{3} \cdot \frac{3}{2} x^{\frac{1}{2}}$

$= 2\sqrt{2} \cdot \sqrt{x}$

$\int_0^1 \sqrt{1 + (2\sqrt{2}\sqrt{x})^2}\, dx = \frac{13}{6}$

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A vertical tangent

Find the length of the curve $y = \sqrt[3]{x}$ between (-8,-2) and (8,2)

$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} = *$ | $x = 0$

$\frac{dy}{dx} = *$

$x = y^3$

$\frac{dx}{dy}$ ok $3y^2$

$\int_{-2}^{2} \sqrt{1 + (3y^2)^2}\, dy$

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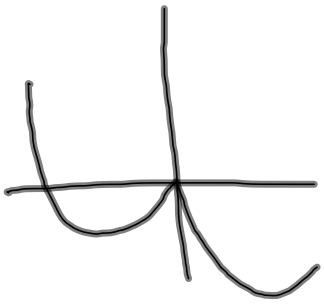
A cusp

E x 4

Find the length of the curve $y = x^2 - 4|x| - x$ from x=-4 to x=4



$$\int_{-4}^{0} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx + \int_{0}^{4} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

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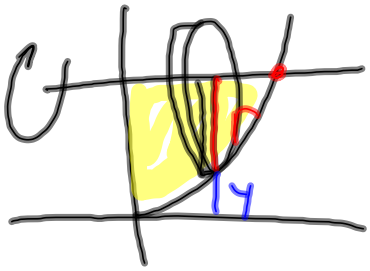
21. $y=\sqrt{2}$ $y=\sec x \tan x$ y axis



$$\int_a^b \pi r^2\,dx$$

$a=0$ $b=.7854$

$r=\sqrt{2}-y$

$r=\sqrt{2}-\sec x \tan x$

$$\int_0^{.7854} \pi\left(\sqrt{2}-\sec x \tan x\right)^2 dx$$

2.301

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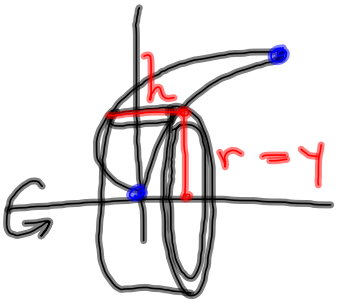
34. $x = \frac{y^4}{4} - \frac{y^2}{2}$ $x = \frac{y^2}{2}$



$\int_c^d 2\pi r h \, dy$

$\int_0^2 2\pi y \left(\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2} \right) \right) dy$

$h = \frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2} \right)$

$\frac{8\pi}{3}$

$c = 0$

$d = 2$

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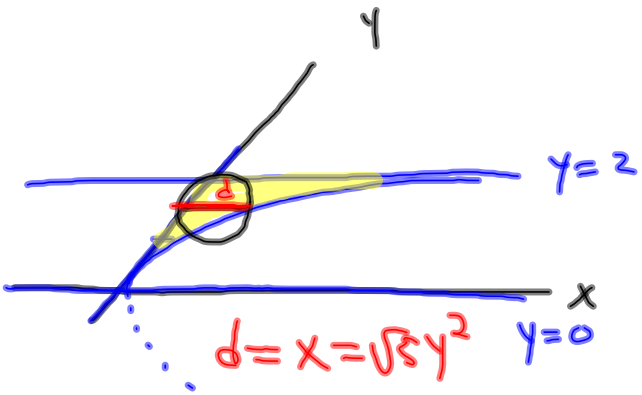
41. $y = 0$ $y = 2$ $x = \sqrt{5}\, y^2$



$d = x = \sqrt{5} y^2$

$r = \frac{\sqrt{5}}{2} y^2$

$c = 0$ $d = 2$

$\int_c^d \pi r^2 \, dy$

$\int_0^2 \pi \left(\frac{\sqrt{5}}{2} y^2 \right)^2 dy$

$= 8\pi$

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